

LOSS OF PRESSURE IN WATER FLOWING THROUGH 90° PIPE BEND

by

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## INTRODUCTION

### Need For Study

The 90 degree bend is perhaps the most frequently used fitting in pipe systems. The pressure losses in such bends are therefore of considerable engineering importance. However, although many investigators have studied the problem, the results which they have obtained have not been satisfactorily correlated. It is evident that a more detailed study is necessary before the problem is placed on a certain and systematic basis. Recent work of Pigott (6) is welcomed, as he furnished a more nearly rational and consistent method of computing loss of head due to bends. Some idea of the amount of experimental work that has been done on flow in bends and the growing appreciation over the years of the different factors that influence the pressure loss in bends may be gained by inspection of previous surveys. The importance of geometrical parameters such as radius ratio,  $R/r$ , bend angle,  $\phi$ , and duct cross-sectional shape was appreciated from the outset. Later the dependence of the pressure loss on the Reynolds number, state of the inlet velocity profile and conditions of the ducting on either side of the bend was recognized.

### Scope of Study

The experiments were conducted in a 2 in. PVC pipe. The water, pumped from a reservoir, entered the pipe under test.

About 100 diameters were used for a calming length between the inlet and the first piezometer tap. The discharge varied from 0.04 to 0.06, 0.10, 0.14, 0.18, 0.22 and 0.26 c.f.s. successively for the horizontal pipe bend test. The same discharges exclusive of the largest one, 0.26 c.f.s., were used for the vertical pipe bend test. Five series of runs were used for obtaining the data for this study. The results of experimental studies to determine the pressure losses for turbulent flow in smooth pipe bends of circular cross section are presented in this thesis.

#### Purpose of Study

The first group of experiments reported here was designed to study the effect of the downstream tangent on the bend losses. The influence of variations of the Reynolds number on the magnitude of the bend loss coefficient was also considered. The 2 in. pipe with the 90 degree bend was used first to determine the pressure lost for the Reynolds numbers ranging from  $2.82 \times 10^4$  to  $1.86 \times 10^5$ . In order to compare the bend loss coefficient of horizontal pipe bends with that of vertical pipe bends, the pipe connected with the bend was first set up in the horizontal plan and then turned to the vertical plan.

The second aspect of the problem considered was the effect on the pressure loss of a variation in downstream transition length. The original pipe has a transition length of about 40 diameters and was reduced in five stages.

## REVIEW OF PREVIOUS INVESTIGATIONS

There has been a great deal of research into pressure losses at bends in pipelines in the past. Most of the early work was of doubtful value owing to shortcomings in the design of the apparatus. Often the length of straight pipe upstream of the bend was insufficient to allow the flow to become steady before the bend was reached. Indeed, some of the early investigators measured the pressure loss between points as close as one or two diameters on either side of the bend. It is also likely that in much of the early work commercial pipes with imperfectly fitting joints were used. Very often the effect of these imperfections would be nearly as great as that of the bends. Therefore, only a few representative previous works are selected.

In 1913, Balch (5) made an experimental study, investigating the relation of the loss of head to the radius of the bend. The experimental apparatus was limited to pipes and bends of circular cross section, 3 in. in diameter, the bends having a turn of 90 degrees, but of various radii. Balch concluded that the total loss of head in bends decreased with an increase in radius until the radius equaled about 4 pipe diameters. For bends with radii greater than 5 pipe diameters the total loss increased with an increase in the length of radius.

In 1938, Beij (1) investigated the pressure losses for fluid flow in 4 in. steel, 90 degree pipe bends of radii from

6 in. to 80 in. The results were discussed in relation to those found by previous investigators under comparable test conditions. For bends having radii of four pipe diameters or less, all the results which were discussed might be correlated on the basis of pipe roughness. No correlation could be obtained for the bends of larger radii.

In 1960, Eastwood and Sarginson (4) made an experimental investigation into the effect of transition curves on the head lost in flow round 90 degree bends in pipelines. Spiral transitions subtending an angle of 45 degrees at the beginning, the end, and the beginning and end of the bend were investigated, as well as transitions subtending an angle of 30 degrees at the beginning and end of the bend. The radii of the circular part of the bends varied between 3 in. and 18 in., while the cross section of the pipe was 2 in. square. In all cases the introduction of a transition curve reduced the head loss by up to 20 percent, but the form of transition that gave the greatest reduction varied with the sharpness of the bend.

In 1960, Martin and Deverson (7) studied the effect of bend outlet conditions on the pressure losses. In an experiment made with a three coil helical pipe (i.e. bend deflection equal  $6\pi$ ) a value was found for the bend deflection corresponding to a complete oscillation of the secondary circulation. From this result a bend radius might be determined for a given pipe diameter such that certain secondary flow conditions were

attained at the outlet of a bend of given deflection. The build up of pressure losses in both the bend itself and in the transition region downstream from the bend was investigated for three values of the bend deflection. Similar calculations were made for flow in a helical pipe at approximately the same Reynolds number.



## THEORETICAL BASIS

## The Nature of Flow in a Bend

Consider a fluid flowing along an initially straight pipe. The velocity profile eventually attains a characteristic form which is independent of distance along the pipe. Conditions are then referred to as fully developed. If now the flow direction is changed the fluid near the axis of the pipe, having the highest velocity, is subjected to a larger centrifugal force than the slow moving fluid in the neighborhood of the pipe wall. This causes a secondary flow to arise, which varies in an oscillatory manner with downstream distance, in which the flow in the center of the pipe moves away from the center of curvature and the fluid in the neighborhood of the wall flows towards the center of curvature. The position of the fluid having maximum velocity is also displaced outwards in an oscillatory manner from the axis of the pipe.

In practice, there are large frictional damping effects present which, provided the bend is sufficiently long for this to happen, completely eliminate the secondary flow oscillations leaving steady secondary flow conditions. In these circumstances a region of fully-developed curved flow is established. The distinctive feature of such a region is that the velocity and pressure distributions over a cross section are independent of distance round the bend. Fig. 1 illustrates diagrammatically fully-developed curved flow in a pipe of circular cross section. Secondary flow effects



occur in pipes of any cross section and since they are generated and sustained by the wall in the plane of the bend, the cross sectional shape has an important bearing on the magnitude of these secondary flow processes.

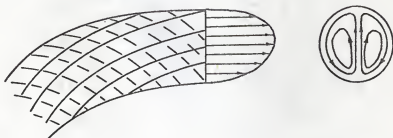


Fig. 1 Secondary flow in a curved pipe

The final phase of the turning process is the return of the flow to that in a straight pipe. Downstream from the bend outlet the rotational energy in the secondary flow is dissipated. Simultaneously a re-distribution of the velocity profile occurs until eventually fully developed flow is attained.

#### The Pressure Loss in a Bend

There are, basically, two common ways of defining the pressure loss in a bend. One method is to define the loss with reference to the change in total pressure; the other is based on the use of static pressure differences.

##### (a) Energy Considerations:

Consider a pipe in which the bend deflection is  $\phi$  and the downstream tangent length is  $l_d$ .

The pressure losses in the system may be calculated from the difference between the values of total pressure at any two arbitrary stations spanning



Fig. 2

the bend. Then the energy flux through any cross section is

$$\int_0^{d/2} \int_0^{2\pi} h \rho v r dr d\theta$$

A mass averaged stagnation pressure at a cross section  $H$  may be defined by the integral

$$H = \frac{4}{\pi d^2 \bar{v}} \int_0^{d/2} \int_0^{2\pi} h v r dr d\theta \quad (1)$$

where  $h = P + \frac{1}{2}\rho(u^2 + v^2 + w^2) = P + \frac{1}{2}\rho q^2$  is the stagnation pressure at the point  $(r, \theta)$

$u, v, w$  are respectively the values of the radial, axial and tangential components of the local velocity,  $q$ .

$P$  is the local static pressure

$\bar{v}$  is the mean axial velocity

As shown in Fig. 2,  $r$  and  $\theta$  are polar coordinates.

At an upstream station, assuming that the upstream tangent is long enough to ensure fully developed turbulent flow there, the velocity profile is symmetrical about the pipe axis and therefore  $h$  is independent of  $\theta$ .

$$H_u = \frac{8}{\pi d^2} \int_0^{d/2} h v r dr$$

At a downstream station, a surface traverse may be made which will give  $h = h(r, \theta)$ . A numerical value for  $H_d$  may therefore be determined.

The total pressure drop across the bend is  $(H_u - H_d)$  and equal to the energy dissipated by friction.

If for a moment the details of the mass-averaging process are ignored then the energy at a station may be divided into three components: pressure energy (P) + axial Kinetic energy (K.E.) + rotational energy (R.E.). Hence the energy dissipated between any two stations, the change of total pressure, is

$$H_u - H_d = P_u - P_d + (K.E.)_u - (K.E.)_d + (R.E.)_u - (R.E.)_d \quad (2)$$

and since  $(R.E.)_u = 0$

$$H_u - H_d = P_u - P_d + (K.E.)_u - (K.E.)_d - (R.E.)_d \quad (3)$$

Now if the energy states are referred to stations in region of fully developed flow, then

$$H_u - H_d = P_u - P_d \quad (4)$$

In such circumstances the loss of energy is equal to the drop in static pressure.

For the configuration under discussion the rotational energy in the downstream tangent is not recoverable as pressure and Kinetic energy and is eventually dissipated by friction. It is therefore usually found more convenient to write equation (3) in the form of a static pressure difference rather than a

total pressure difference. Thus

$$P_u - P_d = (H_u - H_d) - (K.E.)_u + (K.E.)_d + (R.E.)_d \quad (5)$$

The largest contribution to the static pressure loss will, in general, be the dissipation of energy associated with the frictional properties of the fluid,  $(H_u - H_d)$ . From the energy balance expressed in equation (5) it will also be seen that the increase in rotational energy occurs at the expense of the pressure energy. The third source of static pressure change is associated with the changing velocity profile along the pipe.

If energy considerations are used as a basis for defining mean flow properties then a knowledge of both the velocity distribution (axial and rotational) and the variation of the static pressure within the pipe is required. Such information is obviously useful for the detailed study of the flow in bend. On the other hand, for the purpose of calculating the variation of flow properties along a pipe, for instance in system performance calculations, it is generally a disadvantage to have to consider such factors and an alternative approach is required.

#### (b) Momentum Considerations:

A different approach which avoids the necessity of considering the velocity distribution follows from momentum considerations. The force acting at a cross section due to the static pressure is

$$\int_0^{d/2} \int_0^{2\pi} P r dr d\theta$$

and a mean static pressure at a cross section may be defined as

$$\bar{P} = \frac{4}{\pi d^2} \int_0^{d/2} \int_0^{2\pi} P r dr d\theta \quad (6)$$

In order to avoid considering the detailed variation of the static pressure distribution over a cross section the mean static pressure is frequently defined in another way. A number of pressure tappings are made at uniform intervals round the pipe wall and the mean static pressure is simply defined as the arithmetic mean of these values.

The momentum of the flow passing a cross section per unit time is expressed by

$$M = \int_0^{d/2} \int_0^{2\pi} \rho v^2 r dr d\theta$$

According to Newton's second law of motion, the change of momentum per unit of time in the body of water in a flowing pipe is equal to the resultant of all the external forces that are acting on the body. Applying this principle to a pipe containing a bend, the following expression for the momentum change per unit time in the body of water enclosed between any two stations spanning the bend may be written:

$$M_d - M_u = \bar{P}_u - \bar{P}_d - F_f \quad (8)$$

where  $F_f$  is the total external force of friction and resistance acting along the surface of contact between the water and the pipe.

Now if the momentum states are referred to stations in region of fully developed flow, then

$$F_f = \bar{P}_u - \bar{P}_d$$

In such circumstances the loss of bend is

$$\Delta P = \bar{P}_u - \bar{P}_d - \frac{f(x_u + x_d)}{d} \frac{1}{2} \rho v^2$$

The advantage of this method is that a detailed knowledge of the flow is not required and experimental data are readily obtained in a useful form.

A method based on a limited application of this approach together with the familiar concept of the one dimensional equivalent flow will be used in this thesis.

#### (c) Definition of Bend Loss Coefficient:

Close to the bend it is possible for the static pressure differences at a particular station to be larger than the pressure drop between corresponding points on either side of the bend. The definition of the bend loss, referred to such a station, then depends critically on the method selected to define the mean pressure at a cross section. In order to avoid such uncertainties it is advantageous to relate information to selected stations on either side of the bend where the static pressure may be simply, yet unambiguously, defined in terms of the static pressure at the pipe wall. The requirement is met most conveniently by considering stations where the static pressure is constant at the pipe wall.

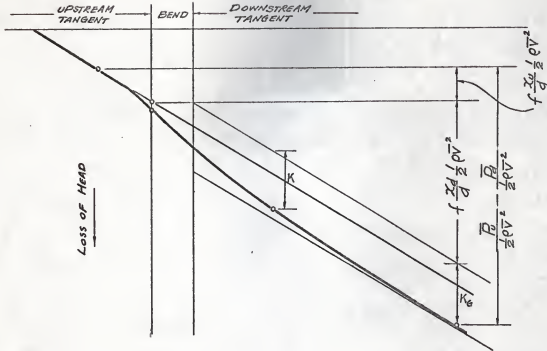


Fig. 3 Definition of the bend loss coefficient

Figure 3 is used to explain the definition of the bend loss coefficient. The pressure loss attributable to a bend,  $\Delta P$ , is determined from the measurements at any two arbitrary stations spanning the bend subject only to the restrictions stated in the previous paragraph. The measured pressure drop may be considered to be made up of two components. One is the pressure drop, which must be defined, that would have existed in the absence of the bend, the other being the pressure drop due to the bend. Here the pressure drop that would have existed in the absence of the bend is defined as the pressure drop corresponding to fully developed flow in a length equal to the straight portions between the measuring stations. This leads to the definition of the bend loss



coefficient,  $K$ , as shown below. An alternative method is to define this quantity as the pressure drop corresponding to fully developed flow in a straight pipe having the same length as the center line of the bend plus tangents. This leads to the definition of the excess or net bend loss coefficient,  $\zeta$ .

Thus

$$K = \frac{\Delta P}{\frac{1}{2}\rho v^2} \quad (9)$$

where 
$$\Delta P = \bar{P}_u - \bar{P}_d - \frac{f(x_u + x_d)}{d} \frac{1}{2}\rho v^2$$

$f$  is the friction factor for fully developed flow in a straight pipe.

$K$  and  $\zeta$  are related by the expression

$$\zeta = K - f \frac{R}{d} \frac{\phi}{57.3}$$

where  $1/57.3$  is the conversion factor to change degrees to radians.

On technical grounds an equally good case can be made for the use of  $K$  or  $\zeta$ . However,  $K$  is slightly easier to use in general system calculations and for this reason it has been selected to present the results of the experimental data.

When the upstream and downstream measurements are made at stations in the regions of fully developed flow  $K$  is then independent of  $x_d/d$  and is called the gross bend coefficient,  $K_G$ .

## EXPERIMENTAL PROCEDURE

## (a) Experimental Apparatus

The pipe bend used in the tests was a commercial PVC 90 degree elbow of 2.067 in. I.D., having a 3.28 in. radius of curvature. The straight pipes, or tangents, connected with this bend were of commercial rigid PVC conduit of 2.067 in. I.D. The pipe joint was made with coupling. A typical arrangement of experimental apparatus is illustrated in Fig. 4. The water, led from a reservoir, entered the pipe under test. About 100 diameters were used for a calming length between the leading end and the first piezometer tap. The manometer connection consisted of four holes drilled at 90 degree intervals in the circumference of the pipe bend and of one hole in the straight pipes. The discharge was measured by a pipe orifice. The flow was controlled by a valve at the leading end of the line. Sufficient care was taken to scrape off the burrs so that the inside edge of each piezometer hole was flush with the surface..

## (b) Preliminary Experiments

There were two preliminary experimental operations: the calibration of the pipe orifice and the determination of pipe friction factor.

## Calibration of the pipe orifice:

The discharge, and consequently the velocity of flow through the pipe line, was measured by means of an annular

Table 1. The Spacing\* of Downstream Piezometers

Piezometer No.		$D_1$	$D_2$	$D_3$	$D_4$	$D_5$
1 <sup>st</sup>	Stage $x_d/d$	0	9	18	27	36
2 <sup>nd</sup>	Stage $x_d/d$	0	9	18	27	
3 <sup>rd</sup>	Stage $x_d/d$	0	9	12	18	
4 <sup>th</sup>	Stage $x_d/d$	0	3	6	9	12
5 <sup>th</sup>	Stage $x_d/d$	0				

\* Spacing is shown in terms of the ratio  $x_d/d$ .

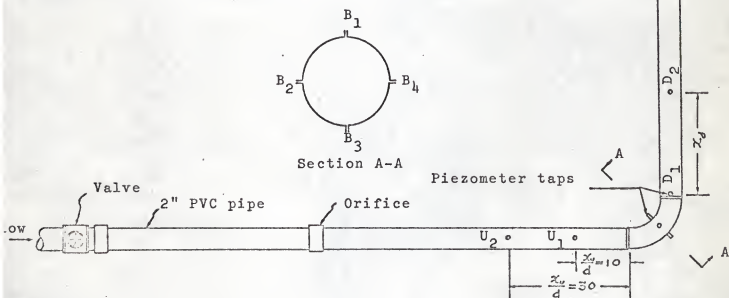


Fig. 4 Experimental Apparatus

orifice shown in Fig. 6. The orifice was placed almost in the middle of upstream pipe for determining the total discharge of the system. It was calibrated over the range of flows. By timing the flow of a certain amount of water into a measuring tank and reading the corresponding head difference of the manometer, the rate of flow could be calculated. The calibration curve is plotted as Fig. 7.

#### Determination of Friction Factor:

The experimental apparatus is shown schematically in Fig. 5. The loss of head from b to d and from d to c could be read from the manometer.

By the Darcy Equation  $h_f = f(L/d)(\bar{v}^2/2g)$ , where  $h_f$  was the head loss in the distance L. Solving for the friction factor it was found  $f = (2gdh_f)/(L\bar{v}^2) = (\pi^2gd^5h_f)/(8LQ^2)$ . Since  $d = 2.067$  in. and  $L/d = 40$  then

$$f = 8.67 \times 10^{-4} h_f / Q^2 \quad (10)$$

Moreover the Reynold's Number,  $N_R = (\bar{v}d/\gamma) = (4Q)/(\pi d\gamma)$ . For a temperature equal to  $70^\circ\text{F}$ ,  $\gamma = 1.05(10^{-5}) \text{ ft}^2/\text{sec}$ . So that

$$N_R = 7.04(10^5)Q \quad (11)$$

From equations (10) and (11), it was necessary to measure only  $h_f$  and  $Q$ , making it possible to plot a curve of  $f$  vs  $N_R$ .

The total number of test points were thirty. The data of the test are presented in the appendix. The test range was from a Reynolds number of  $2.82 \times 10^4$  to  $1.86 \times 10^5$ . The resulting Stanton curve ( $f$  vs  $N_R$  curve) is plotted in Fig. 8.

The curve of Blasius' equation  $f = 0.3164/N_R^{0.25}$  for smooth pipe is also plotted for the purpose of comparison.

(c) Experiments for Obtaining Data

As mentioned above, the experiments were composed of five series of runs, based on the downstream pipe length. At first, the downstream pipe had a transition length of 41.2 diameters. Five piezometer taps were drilled equally spaced along the downstream pipe. The first tap was located at the bend outlet. The spacing between adjacent taps was 9 diameters. The discharge varied from 0.04 to 0.06, 0.10, 0.14, 0.18, 0.22 and 0.26 c.f.s. successively for the horizontal bend test. By adjusting the opening of the control valve on the pump and keeping the head difference of the orifice manometer to a certain value, any particular discharge could be obtained. Once the horizontal bend test was completed, the bend connected with downstream pipe was turned to the vertical plane. The discharge varied from 0.04 to 0.06, 0.10, 0.14, 0.18 and 0.22 c.f.s. successively for the vertical bend test. After finishing the first series of runs, the downstream pipe was reduced successively to 28.4, 19.5, 13.5 and 1.5 diameters. The numbers and the spacing of the piezometer taps were changed in every series of runs. The layout of the piezometer taps in each series of runs is shown in Table 1. In the same manner, both the horizontal and vertical bend losses were tested in every stage. In order to determine the pressure distribution at a cross section along the downstream pipe, in the fourth

series of runs the downstream pipe was rotated 90 degrees each time until turned one cycle. This was approximately equivalent to saying that four piezometer taps were drilled at each cross section at 90 degree intervals around the perimeter.

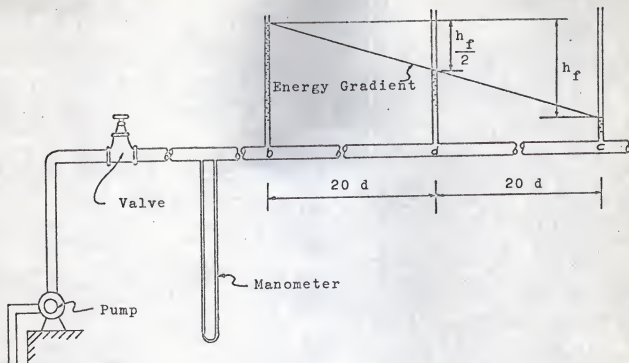


Fig. 5

Experimental Apparatus for Determining Pipe Friction Factor

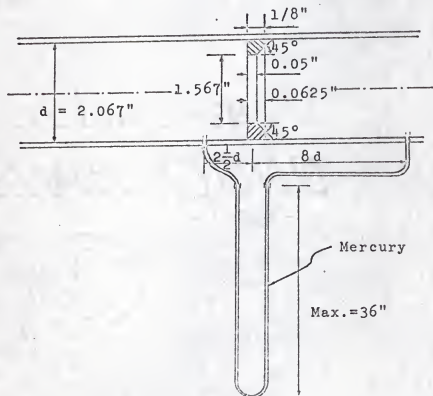


Fig. 6 Orifice Plate and Manometer



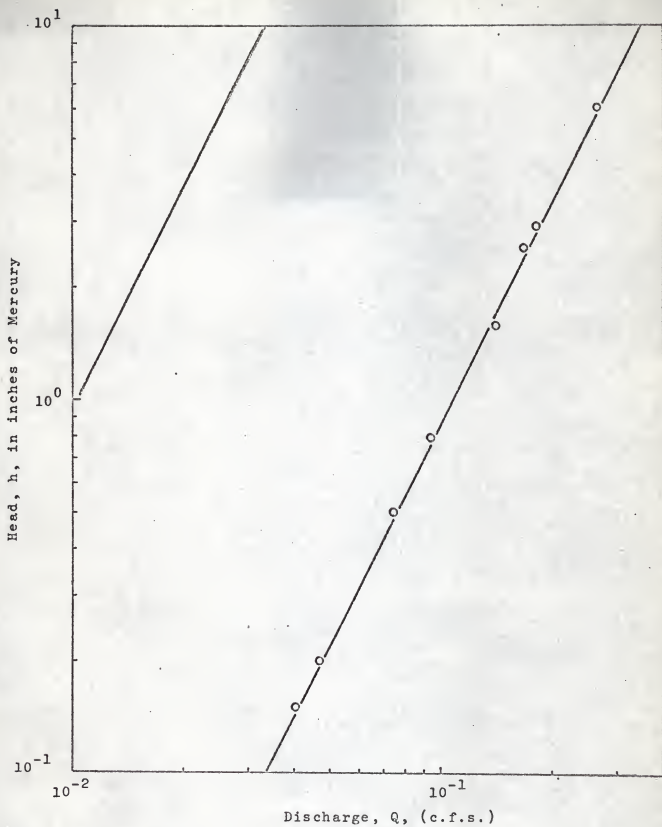


Fig. 7 Orifice Calibration Curve

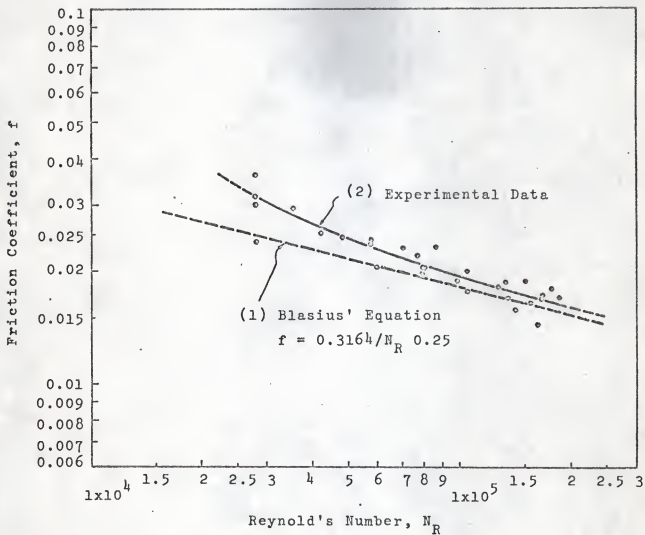


Fig. 8 Stanton Curve for 2 Inch PVC Pipe

## DISCUSSION OF RESULTS

## (a) Resistance Coefficient for Straight Pipe:

The curve obtained from plotting friction factor,  $f$ , against Reynolds number,  $N_R$ , for all runs on the straight pipe is shown as curve 2 in Fig. 8; curve 1 is a graph of Blasius' smooth pipe equation  $f = 0.3164/N_R^{0.25}$ .

It is evident that the value of  $f$  of the 2" PVC pipe is larger than that of the smooth pipe. When the Reynolds number decreased further this tendency became more pronounced.

## (b) Pressure Distribution Round a Bend

In the following discussion it is assumed that the upstream tangent was long enough for establishing the fully developed flow. The data for the upstream piezometers showed that this assumption was fairly accurate. Figures 9 through 15 showed the pressure distribution along the pipeline containing the pipe bend at Reynolds numbers of  $2.82 \times 10^4$ ,  $4.22 \times 10^4$ ,  $7.04 \times 10^4$ ,  $9.85 \times 10^4$ ,  $1.27 \times 10^5$ ,  $1.55 \times 10^5$ ,  $1.86 \times 10^5$  respectively. Observing these figures, the adverse pressure gradients are seen to exist on the outer side of the bend over the first part of the bend and on the inner side of the bend over the last part of the bend. The slope of this gradient is much steeper on the inner wall. The figures also indicate that large pressure gradients arise in the direction of motion as a result of the action of centrifugal force. In Fig. 16 these pressure gradients are clearly seen to extend

about six diameters along the downstream tangent. It may be anticipated that these pressure gradients will also extend a few diameters along the upstream tangent.

(c) Influence of the Downstream Tangent

There is no doubt that the influence of the downstream tangent plays an important role in the bend loss. From Figures 9 through 15, it is evident from inspection of the results for the downstream tangent that as the fluid was forced through a pipe bend the value of  $h/(v^2/2g)$  decreased and the hydraulic gradient changed gradually in a non-linear relationship. This change illustrated that a considerable portion of the energy was lost in the downstream tangent. The flow proceeded downstream with a new modified velocity profile. The effects due to the presence of the bend decreased as the flow moved downstream. But in every case these effects were much reduced beyond  $x_d/d = 25$  and were almost undetectable in more than 36 diameters downstream from the bend. Finally the hydraulic gradient coincided with the gradient in the straight pipe upstream. Eventually the fully developed condition was attained again.

Figure 16 illustrates the pressure distribution at a cross section measured at downstream distances of 0, 3, 6, 9, and 12 diameters respectively. It is evident that the differences in the static pressure at a cross section persisted downstream from the bend exit. However, a constant static pressure at a cross section was obtained at six diameters downstream of

the bend exit. Thus the pressure distribution at a cross section reverted quite quickly as compared with the attainment of a fully developed velocity profile.

(d) Relation of Reynolds Number to Both the Horizontal and Vertical Bend Losses

As mentioned above, the effects due to the presence of the bend were undetectable in more than 36 diameters downstream from the bend. Therefore the bend losses shown in Figures 9 through 15 could be considered as the gross bend losses. Figure 17 is the plot of both the horizontal and vertical gross bend loss coefficients against Reynolds number. In all cases bend losses were determined from the data for the section  $x_u/d = 30$  to  $x_d/d = 36$ . The plotted data indicated that the reduction of  $K_G$  of both the horizontal and vertical bend plane occurred for increasing Reynolds number. The larger the Reynolds number, the less the reduction will be. From Fig. 17 it is also seen that losses in the bend lying in the vertical plane appeared to be slightly larger than that of the bend lying in the horizontal plane at Reynolds numbers of  $2.82 \times 10^4$ ,  $4.22 \times 10^4$  and  $1.55 \times 10^5$ . But on the contrary, the relation was reversed for the Reynolds numbers of  $7.04 \times 10^4$ ,  $9.85 \times 10^4$  and  $1.27 \times 10^5$ . However, the differences of bend loss coefficients between the horizontal and vertical plane did not seem to be a significant amount.

(e) Effect of the Downstream Transition Length

The 2 in. pipe with the 90 degree bend described above was also used to study the effect on the pressure losses of

a variation in downstream transition length. Special care was taken to ensure that inlet conditions were as near as possible identical for each system. The Reynolds number throughout being calculated equal to  $9.85 \times 10^4$ . The variations of  $K$ , defined above by equation (9), with  $l_d/d$  at the points of 0, 9, and 18 diameters downstream from the bend exit are presented in Figures 18, 19, 20 and compared there with the results obtained with the vertical bend operating at the same Reynolds number, where  $l_d$  is the length of the downstream tangent. From Figures 18 and 19 it indicated that as the length of the tangent was reduced, the effect of varying the length of the downstream tangent on the bend loss coefficient of both the horizontal and vertical plane measured at  $x_d/d = 9, 18$ , was almost negligible. From Fig. 20 it was seen that a remarkable change of bend loss coefficient measured at bend exit occurred as the downstream transition length was less than 12 diameters. An attempt has been made in this study to observe the change of the pressure distribution around the bend as the downstream transition length was progressively reduced. Unfortunately, the experiment failed because it was impossible to obtain the same total head in the inlet section in each series of run. It might be anticipated that change of the pressure distribution will occur on the inside of the bend as the transition length decreases.

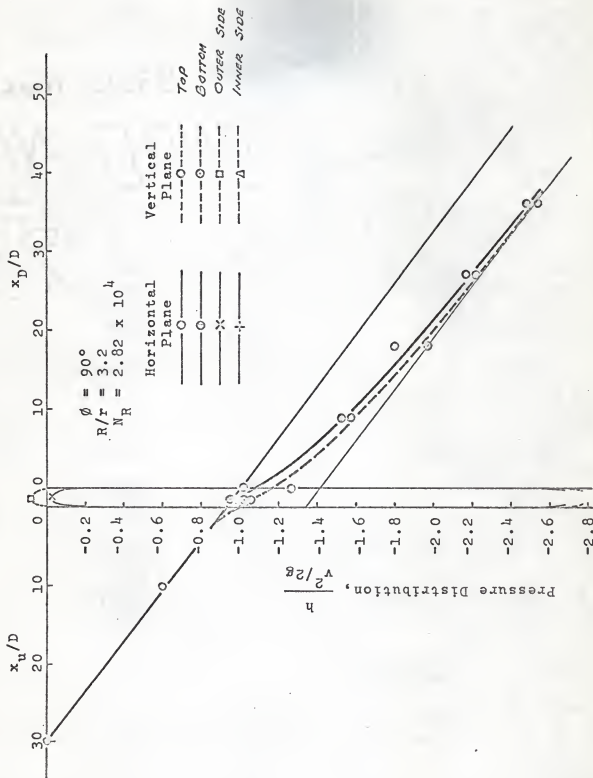


Fig. 9 Pressure distribution along pipeline containing a 90° pipe bend



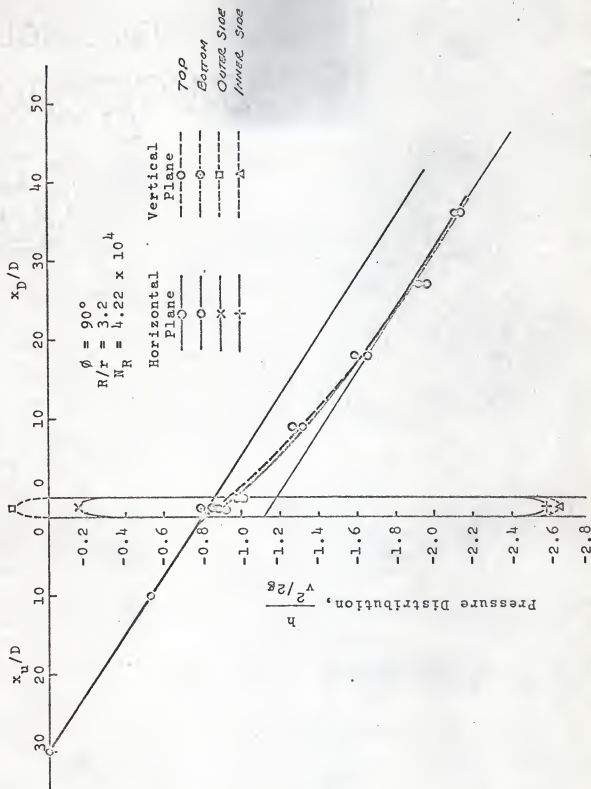


Fig. 10 Pressure distribution along pipeline containing a 90° pipe bend

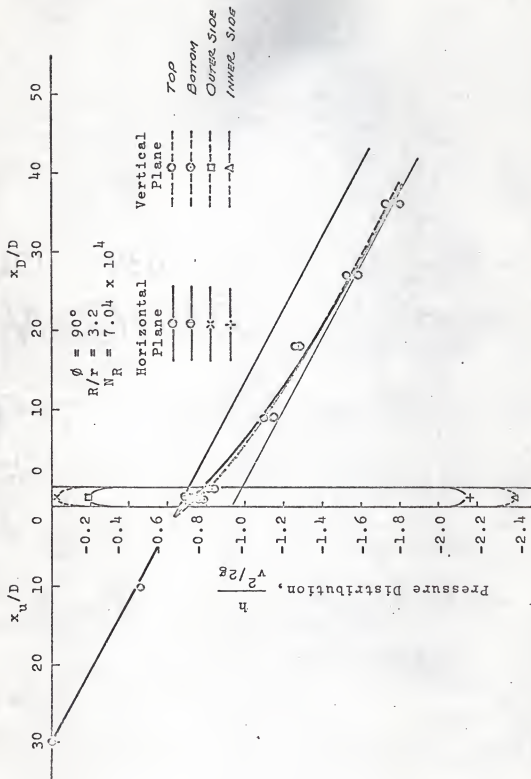


Fig. 11. Pressure distribution along pipeline containing a 90° pipe bend

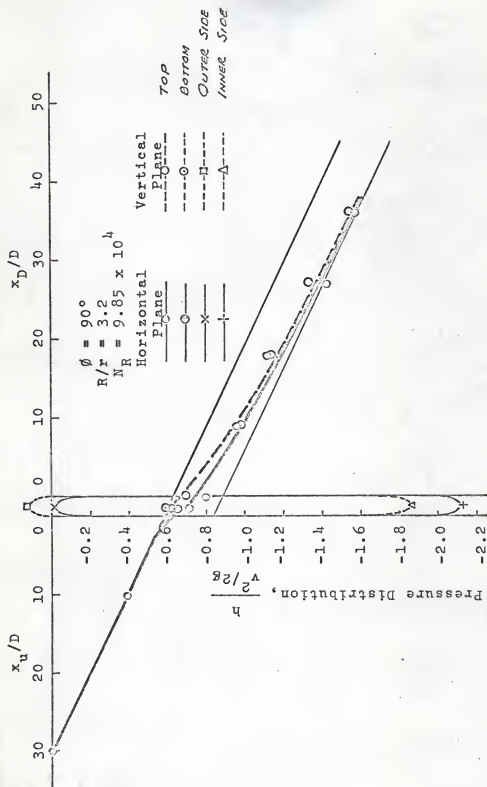


Fig. 12 Pressure distribution along pipeline containing a  $90^\circ$  pipe bend

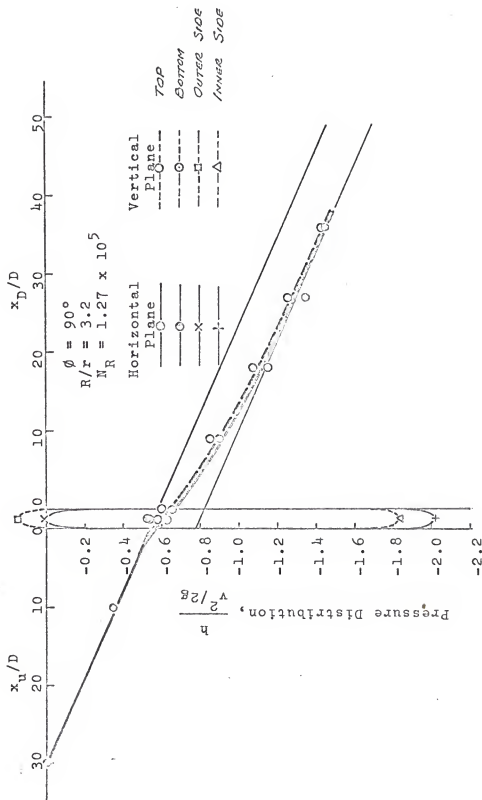


Fig. 13 Pressure distribution along pipeline containing a 90° pipe bend

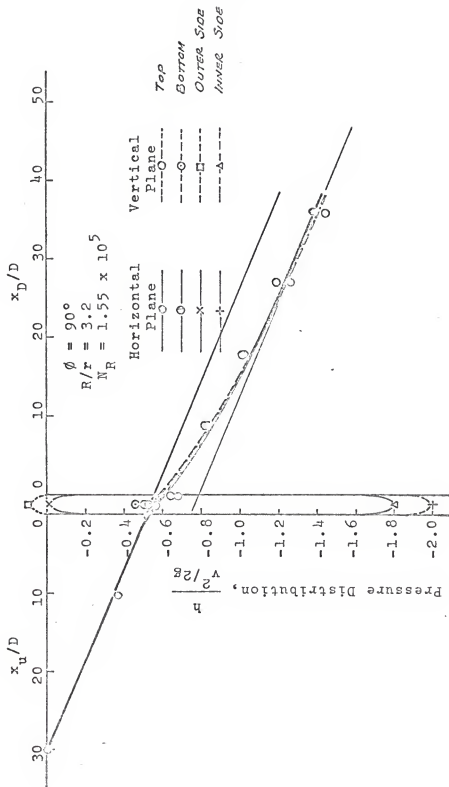


Fig. 14 Pressure distribution along pipeline containing a 90° pipe bend

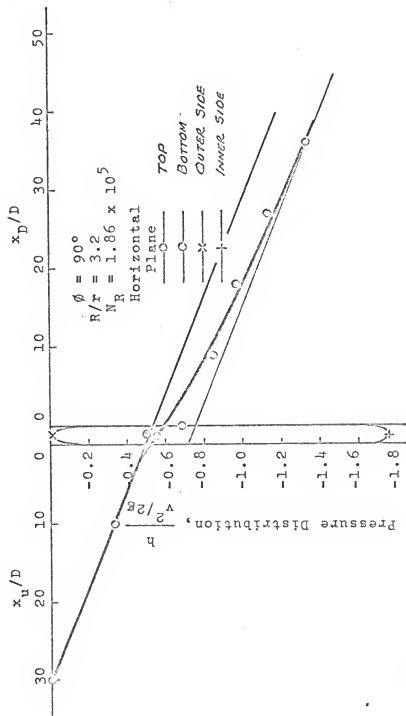


Fig. 15 Pressure distribution along pipeline containing a  $90^\circ$  pipe bend

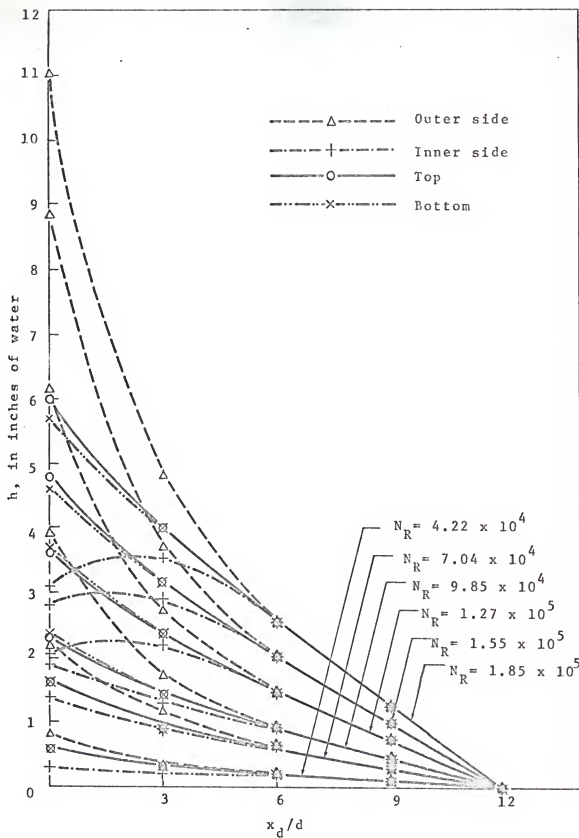


Fig. 16 Variation of the static pressure at a cross section with downstream distance



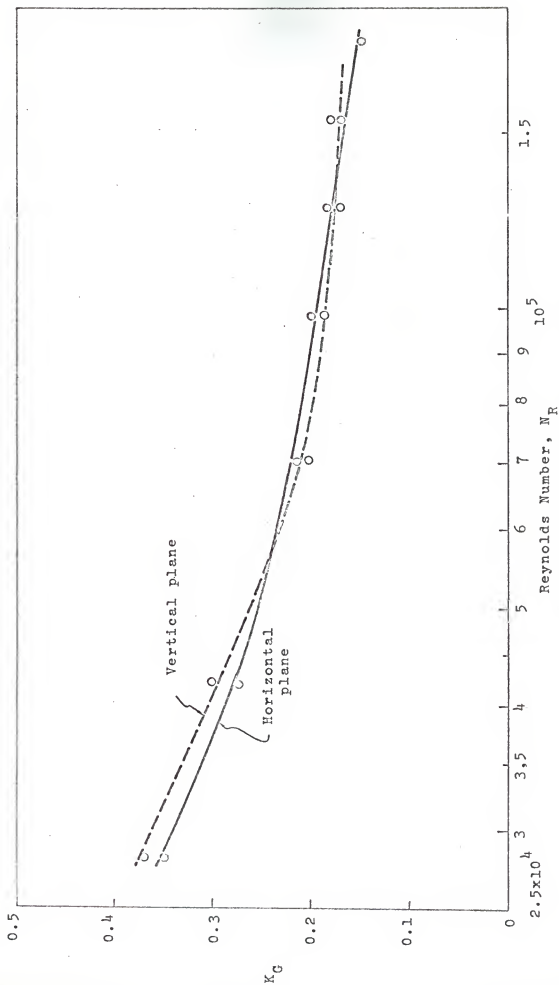


Fig. 17 Variation of gross bend loss coefficient with Reynolds number

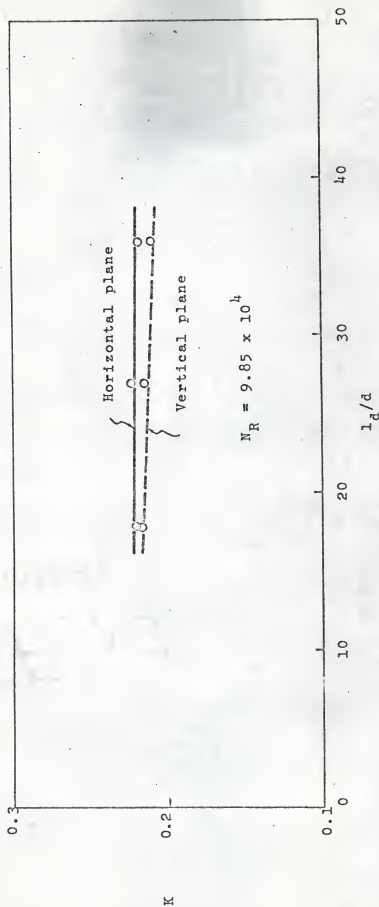


Fig. 18 Variation of bend loss coefficient with transition length (evaluated 18 diameters downstream of bend outlet)

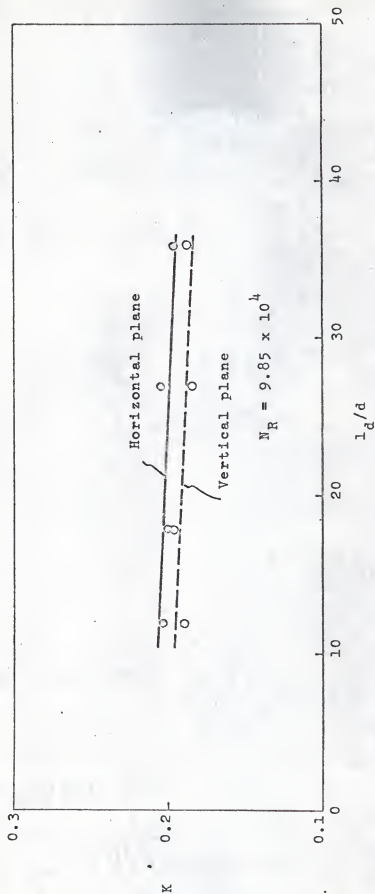


Fig. 19 Variation of bend loss coefficient with transition length  
(evaluated 9 diameters downstream of bend outlet)

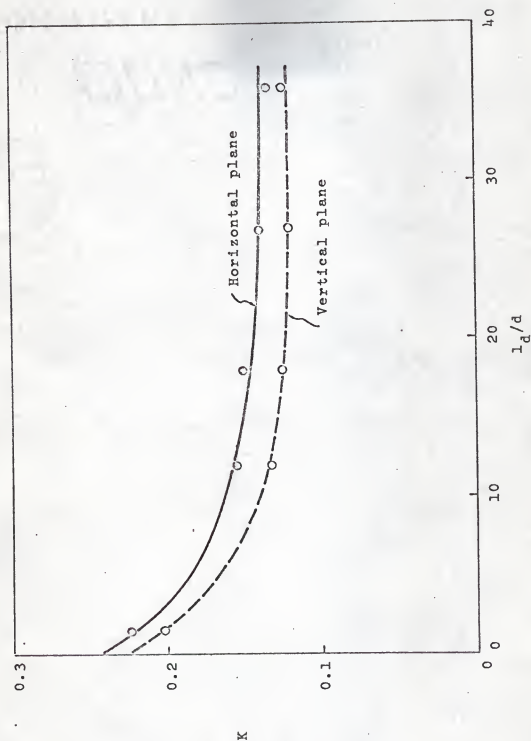


Fig. 20 Variation of bend loss coefficient with transition length (evaluated at bend outlet)

## CONCLUSION

The flow in bends is complicated. In assessing pressure losses in a bent pipe, the bend geometry, duct cross sectional shape and Reynolds number are important as well as both the length of the tangents and the position of the pressure measurements.

It was found that large pressure gradients are created in the direction of motion around the bend and extend a few diameters along the tangents and are not confined solely to the bend.

It was found that the differences in the static pressure at a cross section persisted downstream from the bend exit and extended six diameters for the bend having 3.28 in. radius of curvature.

The bend loss coefficient,  $K$ , depends on the position of the pressure measurements for  $x_d/d$  less than 36 and is independent of the position of the pressure measurements for  $x_d/d$  greater than 36.

It was found that a reduction of  $K_G$  for the pipe bend lying both in the horizontal and vertical plane occurred for increasing Reynolds number  $N_R$ . The difference of  $K_G$  between horizontal and vertical plane did not seem to be a significant amount.

When the tangent has certain value of  $l_d/d$  (less than 12 diameters), for such short tangent,  $K$  depends on both  $x_d/d$  and  $l_d/d$ .

In general, the values for loss coefficient for standard  $90^\circ$  bends range from 0.15 to 0.5.

## RECOMMENDATIONS FOR FURTHER STUDY

The following are recommended as subjects for further investigation.

Further research is needed to observe the change of the pressure distribution around the bend as the downstream transition length is progressively reduced. In doing so, it is necessary to have the same total head in the inlet section in each series of run.

Since the pressure loss in bends is influenced by many factors, further study is also needed to determine the effects of bend deflection, radius ratio, duct cross sectional shape on bend resistance.

## ACKNOWLEDGMENTS

The writer wishes to express his sincere appreciation and deepest gratitude for the guidance, instruction and review given by Dr. Richard M. Haynie, Department of Civil Engineering, Kansas State University.

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## APPENDICES

## APPENDIX I

## Notations

$d$ :	diameter of the main pipe
$f$ :	friction factor for a straight pipe
$F_f$ :	the total external force of friction and resistance acting along the surface of contact between the water and the pipe
$g$ :	gravitational acceleration
$h$ :	the stagnation pressure
$H$ :	the averaged stagnation pressure at a cross section
$h_f$ :	the head loss between two stations
$K$ :	bend loss coefficient
$K_G$ :	gross bend loss coefficient
$K.E.$ :	axial Kinetic energy at a cross section
$l_d$ :	length of the downstream tangent
$M$ :	total momentum at a cross section per unit time
$N_R$ :	Reynolds number
$P$ :	the local static pressure
$\bar{P}$ :	mean static pressure at a cross section
$Q$ :	total rate of flow
$r$ :	radius of the main pipe
$R$ :	radius of curvature of the bend
$R.E.$ :	rotational energy at a cross section
$u$ :	the value of the radial component of the local velocity
$v$ :	local axial velocity
$\bar{v}$ :	the mean axial velocity

w: local tangential velocity  
x: distance measured along tangent from bend entry or exit  
y: net bend loss coefficient  
 $\theta$ : polar coordinate  
 $\gamma$ : viscosity  
 $\rho$ : density of water  
 $\phi$ : bend deflection

suffixes

d: downstream from bend exit  
u: upstream of bend entry

Appendix II Test data\* for determining the bend loss coefficient in the first stage  
 $l/d = 41.2$

Piezometer No. or Discharge	U <sub>1</sub>		U <sub>2</sub>		D <sub>1</sub>		D <sub>2</sub>		D <sub>3</sub>		D <sub>4</sub>		D <sub>5</sub>	
	H	V	H	V	H	V	H	V	H	V	H	V	H	V
0.04 cfs	1.50	1.85	1.25	97.60	1.05	97.40	0.85	97.25	0.65	97.10	0.50	96.95		
	1.65	1.70	1.30	97.65	1.10	97.40	1.00	97.35	0.60	97.00	0.50	96.80		
	1.70	1.95	1.45	97.60	0.85	97.55	0.80	97.35	0.60	97.15	0.65	97.10		
	1.45	1.80	1.15	97.40	0.95	97.20	0.85	97.10	0.85	97.05	0.55	96.95		
	2.70	3.35	2.10	98.55	1.70	98.20	1.35	97.70	0.95	97.30	0.70	97.15		
0.06 cfs	2.65	3.40	2.05	98.70	1.75	98.00	1.30	97.75	0.85	97.25	0.75	97.00		
	2.70	3.45	2.00	98.60	1.70	98.25	1.30	97.70	0.70	97.25	0.75	97.20		
	2.90	3.20	2.25	98.45	1.85	98.15	1.45	97.85	0.85	97.45	0.90	97.20		
	5.90	7.40	4.65	101.00	3.60	99.95	2.85	99.50	2.10	98.45	1.35	97.95		
	5.95	7.45	4.25	101.15	3.65	99.70	2.95	99.40	2.10	98.20	1.00	97.80		
0.10 cfs	5.90	7.75	4.20	100.85	3.65	100.10	2.95	99.50	2.00	98.65	1.25	98.10		
	5.80	7.05	4.60	101.10	3.40	99.85	2.70	99.65	2.20	98.65	1.50	97.85		
	10.50	13.10	7.75	105.45	6.65	103.40	5.70	102.35	3.55	101.05	2.45	99.55		
	10.50	13.15	7.75	105.40	6.50	103.40	5.70	102.15	3.50	101.05	2.40	99.55		
	10.40	13.00	7.90	105.60	6.60	103.65	5.65	102.30	3.50	101.15	2.40	99.40		
0.14 cfs	10.70	13.45	7.65	105.60	6.50	103.20	5.75	102.20	3.55	101.15	4.55	99.45		

\*The data are the pressure heads, measured in inches, at the stations designated.

Test data\* for determining the bend loss coefficient in the first stage  $l_d/d = 41.2$

Piezometer No. Designation of Bend Discharge	U <sub>1</sub>		U <sub>2</sub>		D <sub>1</sub>		D <sub>2</sub>		D <sub>3</sub>		D <sub>4</sub>		D <sub>5</sub>	
	H	V	H	V	H	V	H	V	H	V	H	V	H	V
0.18 cfs	16.40	20.40	13.20	111.85	9.95	109.15	8.40	105.85	5.35	104.70	4.05	102.25	102.25	
	16.40	20.50	13.15	111.85	9.90	109.15	8.35	105.80	5.35	104.75	4.00	102.00	102.00	
	16.45	20.50	13.25	111.90	9.90	109.20	8.40	105.80	5.40	104.75	4.15	102.25	102.25	
	16.25	20.45	13.20	111.85	9.95	109.10	8.40	105.75	5.45	104.75	4.25	102.15	102.15	
0.22 cfs	24.35	30.10	18.75	118.55	16.25	115.70	13.10	112.35	9.00	109.35	5.95	105.65	105.65	
	24.30	30.10	18.80	118.55	16.20	115.75	13.10	112.30	9.00	109.25	5.80	105.65	105.65	
	24.30	30.25	18.75	118.65	16.25	115.60	13.15	112.30	9.10	109.25	5.90	105.60	105.60	
	24.35	30.15	18.75	118.60	16.10	115.60	13.15	112.40	9.05	109.10	5.90	105.65	105.65	
0.26 cfs	31.00	38.70	23.05		17.95		16.10		11.45		7.55			
	31.05	38.70	23.10		18.00		16.10		11.40		7.45			
	31.05	38.65	23.10		18.00		16.00		11.40		7.40			
	31.05	38.60	23.10		17.90		16.05		11.35		7.40			

\* The data are the pressure heads, measured in inches, at the stations designated.

Test data\* for determining the bend loss coefficient in the second stage  $l/d = 28.4$

Piezometer No. Orientation of Bend Discharge	U <sub>1</sub>		U <sub>2</sub>		D <sub>1</sub>		D <sub>2</sub>		D <sub>3</sub>		D <sub>4</sub>	
	H	V	H	V	H	V	H	V	H	V	H	V
0.04 cfs	2.15	2.50	1.90	73.20	1.60	73.05	1.45	72.75	1.25	72.60		
	2.15	2.45	1.95	73.20	1.65	73.05	1.40	72.75	1.15	72.60		
	2.10	2.45	1.90	73.15	1.65	73.10	1.45	72.70	1.20	72.65		
	2.10	2.65	1.85	73.25	1.70	73.15	1.40	72.75	1.20	72.60		
	2.75	3.40	2.20	73.40	1.85	73.00	1.45	72.60	1.05	72.30		
0.06 cfs	2.75	3.45	2.20	73.40	1.90	73.00	1.55	72.60	1.00	72.35		
	2.75	3.45	2.15	73.40	1.80	73.05	1.50	72.50	1.00	72.40		
	2.65	3.45	2.10	73.45	1.80	73.10	1.50	72.50	1.10	72.45		
	5.60	7.10	4.35	76.25	3.45	75.25	2.65	74.65	1.70	73.65		
	5.55	7.10	4.35	76.20	3.40	75.15	2.75	74.60	1.70	73.50		
0.10 cfs	5.60	7.05	4.30	76.20	3.40	75.15	2.75	74.60	1.65	73.50		
	5.60	7.20	4.30	76.15	3.50	75.10	2.70	74.60	1.70	73.60		
	9.35	11.95	6.70	80.25	5.25	78.60	4.15	77.25	2.80	75.95		
	9.20	11.85	6.75	80.25	5.20	78.60	4.10	77.20	2.85	75.90		
	9.20	12.00	6.65	80.20	5.20	78.65	4.10	77.20	2.85	75.90		
0.14 cfs	9.30	11.90	6.70	80.15	5.20	78.60	4.15	77.10	2.85	75.95		

\* The data are the pressure heads, measured in inches, at the stations designated.



Test data\* for determining the bend loss coefficient in the second stage  $l_1/d = 28.4$

PIEZOMETER No. ORIENTATION OF BEND Discharge	U <sub>1</sub>		U <sub>2</sub>		D <sub>1</sub>		D <sub>2</sub>		D <sub>3</sub>		D <sub>4</sub>	
	H	V	H	V	H	V	H	V	H	V	H	V
0.18 cfs	14.20	18.20	10.40	85.70	7.70	83.35	6.15	82.00	3.50	79.00		
	14.20	18.35	10.40	85.75	7.75	83.40	6.10	82.00	3.50	79.00		
	14.15	18.20	10.45	85.70	7.75	83.40	6.20	82.05	3.45	78.90		
	14.15	18.25	10.45	85.70	7.65	83.40	6.20	82.00	3.40	78.90		
0.22 cfs	19.85	25.60	14.60	93.10	11.75	90.60	8.60	86.95	4.60	83.10		
	19.80	25.65	14.60	93.10	11.70	90.50	8.65	86.80	4.60	83.10		
	19.80	25.60	14.65	93.20	11.70	90.45	8.60	86.80	4.65	83.10		
	19.80	25.70	14.60	93.20	11.75	90.60	8.70	86.90	4.60	83.15		
0.26 cfs	25.65	33.35	18.40		12.60		10.75		6.05			
	25.60	33.30	18.25		12.65		10.80		6.05			
	25.60	33.30	18.40		12.60		10.85		6.05			
	25.70	33.30	18.30		12.60		10.90		6.10			

\*The data are the pressure heads, measured in inches, at the stations designated.

Test data\* for determining the bend loss coefficient in the third stage  $l_d/d = 19.5$

Pitometer No. Or Bend Discharge	U <sub>1</sub>		U <sub>2</sub>		D <sub>1</sub>		D <sub>2</sub>		D <sub>3</sub>		D <sub>4</sub>	
	H	V	H	V	H	V	H	V	H	V	H	V
0.04 cfs	2.00	2.35	1.75	54.10	1.50	53.90	1.45	53.80	1.25	53.70	1.25	53.70
	2.05	2.30	1.75	54.15	1.45	53.95	1.40	53.75	1.25	53.70	1.25	53.70
	2.10	2.30	1.75	54.10	1.40	54.00	1.45	53.70	1.20	53.75	1.20	53.75
	2.10	2.30	1.80	54.20	1.40	54.00	1.40	53.70	1.15	53.75	1.15	53.75
0.06 cfs	2.55	3.20	1.95	54.25	1.55	53.85	1.40	53.75	1.25	53.55	1.25	53.55
	2.40	3.20	1.90	54.20	1.50	53.85	1.40	53.75	1.25	53.50	1.25	53.50
	2.45	3.25	1.85	54.30	1.50	54.00	1.35	53.90	1.30	53.50	1.30	53.50
	2.45	3.10	1.80	54.30	1.55	53.90	1.50	53.90	1.35	53.70	1.35	53.70
0.10 cfs	4.55	6.05	3.40	56.80	2.30	55.65	2.15	55.60	1.40	55.15	1.40	55.15
	4.60	6.05	3.40	56.75	2.35	55.50	2.20	55.65	1.40	55.20	1.40	55.20
	4.55	6.10	3.45	56.70	2.35	55.60	2.20	55.60	1.40	55.25	1.40	55.25
	4.55	6.10	3.40	56.70	2.30	55.65	2.25	55.70	1.45	55.25	1.45	55.25
0.14 cfs	7.50	10.10	4.95	60.05	3.65	58.70	2.85	58.20	2.30	57.25	2.30	57.25
	7.40	10.10	4.95	60.00	3.70	58.75	2.85	58.15	2.35	57.30	2.35	57.30
	7.40	10.15	4.80	60.05	3.75	58.90	2.90	58.20	2.40	57.30	2.40	57.30
	7.40	10.15	4.85	60.15	3.80	58.75	2.90	58.25	2.30	57.30	2.30	57.30

\*The data are the pressure heads, measured in inches, at the stations designated.

Test data\* for determining the bend loss coefficient in the third stage  $l/d = 19.5$

PIEZOMETER NO. ORIENTATION OF BEND Discharge	U <sub>1</sub>		U <sub>2</sub>		D <sub>1</sub>		D <sub>2</sub>		D <sub>3</sub>		D <sub>4</sub>	
	H	V	H	V	H	V	H	V	H	V	H	V
0.18 cfs	11.25	15.30	7.60	64.65	4.80	61.95	3.90	61.85	3.15	60.40		
	11.20	15.30	7.50	64.60	4.85	61.90	3.85	61.80	3.15	60.45		
	11.20	15.35	7.55	64.65	4.80	61.90	3.80	61.70	3.20	60.40		
	11.25	15.40	7.55	64.70	4.70	61.90	3.80	61.80	3.10	60.50		
	15.80	21.55	10.70	70.60	7.20	68.25	5.55	66.55	4.55	63.60		
0.22 cfs	15.80	21.40	10.65	70.60	7.25	68.20	5.60	66.50	4.40	63.65		
	15.85	21.50	10.60	70.65	7.10	68.20	5.50	66.50	4.60	63.60		
	15.85	21.50	10.70	70.50	7.25	68.30	5.55	66.70	4.50	63.70		
	20.65	28.35	13.65		9.45		7.85		3.85			
	20.70	28.40	13.60		9.40		7.80		3.85			
0.26 cfs	20.70	28.40	13.60		9.45		7.80		3.80			
	20.70	28.30	13.60		9.50		7.80		3.70			

\*The data are the pressure heads, measured in inches, at the stations designated.

Test data\* for determining the bend loss coefficient in the fourth stage  $l_d/d = 13.5$

Pressure No. Orientation of Bend Discharge	U <sub>1</sub>		U <sub>2</sub>		D <sub>1</sub>		D <sub>2</sub>		D <sub>3</sub>		D <sub>4</sub>		D <sub>5</sub>	
	H	V	H	V	H	V	H	V	H	V	H	V	H	V
0.04 cfs	1.95	2.30	1.70	41.05	1.60	40.95	1.55	40.90	1.50	40.75	1.35	40.70	1.35	40.70
	1.90	2.35	1.70	41.10	1.60	41.00	1.50	40.95	1.40	40.70	1.30	40.75	1.30	40.75
	2.00	2.30	1.70	41.15	1.70	40.90	1.60	41.00	1.45	40.75	1.35	40.70	1.35	40.70
	1.90	2.35	1.65	41.10	1.75	40.90	1.60	41.00	1.40	40.80	1.30	40.85	1.30	40.85
	2.20	2.85	1.70	41.20	1.45	41.05	1.30	40.85	1.25	40.70	1.10	40.65	1.10	40.65
0.06 cfs	2.20	2.90	1.75	41.20	1.40	41.10	1.40	40.90	1.25	40.80	1.10	40.60	1.10	40.60
	2.15	2.90	1.70	41.20	1.45	41.00	1.45	41.00	1.20	40.85	1.15	40.80	1.15	40.80
	2.10	2.80	1.80	41.05	1.40	41.10	1.65	41.05	1.40	40.75	1.20	40.75	1.20	40.75
	4.25	5.75	3.15	43.15	2.85	42.90	2.30	42.50	1.95	42.15	1.60	42.00	1.60	42.00
	4.30	5.80	3.10	43.10	2.85	42.95	2.40	42.50	1.90	42.10	1.60	42.00	1.60	42.00
0.10 cfs	4.30	5.85	3.15	43.10	2.80	42.90	2.45	42.45	2.00	42.10	1.65	42.10	1.65	42.10
	4.15	5.70	3.20	43.20	2.80	42.95	2.40	42.40	2.00	42.10	1.65	42.20	1.65	42.20
	6.45	9.05	3.80	46.25	3.25	45.55	3.15	45.15	2.65	45.05	1.85	44.40	1.85	44.40
	6.45	9.10	3.85	46.20	3.20	45.60	3.10	45.10	2.60	45.10	1.80	44.30	1.80	44.30
	6.40	9.05	3.80	46.25	3.20	45.60	3.15	45.20	2.75	45.05	1.90	44.35	1.90	44.35
0.14 cfs	6.40	9.05	3.80	46.25	3.30	45.60	3.20	45.20	2.60	45.20	1.90	44.35	1.90	44.35

\*The data are the pressure heads, measured in inches, at the stations designated.

Test data\* for determining the bend loss coefficient in the fourth stage  $1/d = 13.5$

PIEZOMETER NO. ORIENTATION OF BEND DISCHARGE	U <sub>1</sub>		U <sub>2</sub>		D <sub>1</sub>		D <sub>2</sub>		D <sub>3</sub>		D <sub>4</sub>		D <sub>5</sub>	
	H		H		H	V	H	V	H	V	H	V	H	V
0.18 cfs	9.85	13.80	6.25	50.85	5.00	49.95	4.75	49.40	4.00	47.85	2.65	47.30		
	9.90	13.90	6.10	50.80	5.10	49.80	4.75	49.50	4.00	47.80	2.70	47.20		
	9.70	13.95	6.15	50.85	5.15	49.80	4.70	49.30	3.90	47.80	2.70	47.25		
	9.70	13.90	6.20	50.80	5.15	49.70	4.70	49.35	3.95	47.80	2.75	47.20		
0.22 cfs	13.90	19.65	8.95	55.60	7.60	54.40	7.30	53.40	5.95	53.05	4.10	50.75		
	13.85	19.65	8.90	55.60	7.65	54.40	7.25	53.40	5.90	53.10	4.10	50.80		
	14.00	19.60	8.95	55.65	7.70	54.35	7.25	53.45	5.90	53.10	4.20	50.85		
	13.90	19.65	8.80	55.60	7.65	54.20	7.20	53.40	5.90	53.15	4.25	50.80		
0.26 cfs	16.75	24.45	9.55		7.45		6.05		5.55		3.95			
	16.70	24.40	9.60		7.45		6.10		5.60		3.80			
	16.75	24.40	9.65		7.40		6.10		5.65		4.00			
	16.80	24.30	9.70		7.30		6.15		5.65		3.85			

\* The data are the pressure heads, measured in inches, at the stations designated.

Test data\* for determining the bend loss coefficient in the first stage  $l_d/d = 41.2$

PIEZOMETER NO. ORIENTATION OF BEND DISCHARGE	B <sub>1</sub>		B <sub>2</sub>		B <sub>3</sub>		B <sub>4</sub>	
	H	V	H	V	H	V	H	V
0.04 cfs	1.35	97.70	0.25	96.30	1.35	97.70	1.85	98.10
	1.30	97.60	0.25	96.30	1.30	97.70	1.85	98.10
	1.35	97.60	0.25	96.35	1.30	97.75	1.80	98.15
	1.25	97.75	0.25	96.30	1.35	97.65	1.80	98.20
	2.20	98.70	0.15	96.55	2.30	98.80	3.15	99.55
0.06 cfs	2.20	98.70	0.15	96.45	2.30	98.85	3.20	99.60
	2.15	98.75	0.20	96.45	2.35	98.90	3.15	99.65
	2.20	98.70	0.20	96.45	2.30	98.90	3.20	99.60
	4.90	101.30	-0.05	96.90	5.00	101.20	7.35	104.60
0.10 cfs	4.90	101.25	-0.05	96.80	5.10	101.10	7.35	104.65
	4.80	101.30	-0.10	96.80	5.10	101.15	7.40	104.60
	4.85	101.30	-0.10	96.85	5.05	101.20	7.45	104.50
	8.35	105.95	-1.05	97.50	8.75	106.10	12.95	111.00
0.14 cfs	8.30	105.90	-1.10	97.50	8.70	106.10	12.90	111.05
	8.40	106.00	-1.15	97.60	8.75	106.15	13.00	111.00
	8.45	106.00	-1.15	97.65	8.75	106.15	13.00	111.20
	13.40	112.05	-2.15	98.10	14.50	111.50	20.75	120.40
0.18 cfs	13.40	112.10	-2.05	98.05	14.35	111.45	20.70	120.30
	13.45	112.10	-2.05	98.10	14.30	111.50	20.75	120.35
	13.40	112.15	-2.05	98.20	14.40	111.50	20.70	120.35
	20.45	120.70	-2.25	99.15	20.05	121.35	29.55	131.05
0.22 cfs	20.40	120.60	-2.25	99.25	20.10	121.40	29.50	131.10
	20.45	120.60	-2.10	99.15	20.10	121.30	29.50	131.15
	20.40	120.75	-2.20	99.10	20.05	121.30	29.50	131.10
	25.85		-2.35		27.05		38.30	
0.26 cfs	25.80		-2.40		27.15		38.25	
	25.80		-2.40		27.10		38.20	
	25.70		-2.30		27.10		38.35	

\* The data are the pressure heads, measured in inches, at the stations designated.

Test data\* for determining the bend loss coefficient in the second stage  $l_d/d = 28.4$

PIEZOMETER NO. ORIENTATION OF BEND DISCHARGE	$B_1$		$B_2$		$B_3$		$B_4$	
	H	V	H	V	H	V	H	V
0.04 cfs	2.00	73.45	0.85	71.40	1.90	73.05	2.45	73.50
	2.00	73.45	0.80	71.35	1.90	73.00	2.40	73.50
	2.05	73.40	0.80	71.30	1.85	73.10	2.45	73.45
	2.00	73.40	0.85	71.35	1.90	73.10	2.40	73.50
	2.40	73.50	0.41	71.35	2.40	73.50	3.40	74.75
0.06 cfs	2.45	73.50	0.40	71.30	2.45	73.45	3.45	74.80
	2.40	73.50	0.45	71.30	2.45	73.40	3.45	74.70
	2.45	73.50	0.40	71.35	2.40	73.45	3.45	74.70
	4.65	76.55	-0.35	71.55	4.70	76.55	7.15	78.70
0.10 cfs	4.60	76.50	-0.35	71.50	4.75	76.50	7.20	78.80
	4.65	76.65	-0.40	71.65	4.75	76.60	7.15	78.80
	4.60	76.60	-0.45	71.60	4.70	76.70	7.15	78.75
	7.45	80.50	-1.05	71.75	7.70	80.70	13.05	84.90
0.14 cfs	7.50	80.45	-1.05	71.80	7.70	80.75	13.05	84.95
	7.45	80.55	-1.10	71.80	7.75	80.60	13.10	84.90
	7.60	80.30	-1.10	71.70	7.65	80.65	13.10	84.90
	11.65	86.80	-1.65	72.15	11.90	87.35	20.65	94.20
0.18 cfs	11.60	86.65	-1.70	72.20	11.95	87.40	20.60	94.15
	11.65	86.70	-1.70	72.20	11.85	87.40	20.60	94.10
	11.75	86.65	-1.60	72.15	11.80	87.30	20.75	94.05
	16.00	93.25	-1.90	72.45	16.90	95.00	28.90	103.95
0.22 cfs	16.05	93.30	-1.95	72.40	16.80	95.05	28.80	103.90
	16.00	93.20	-1.85	72.40	16.80	95.15	28.75	103.80
	16.10	93.20	-1.80	72.35	16.85	95.10	28.70	103.80
	21.00		-2.00		21.90		33.35	
0.26 cfs	21.05		-2.00		21.85		33.40	
	20.95		-2.05		21.80		33.40	
	21.00		-2.10		21.90		33.30	

\* The data are the pressure heads, measured in inches, at the stations designated.



Test data\* for determining the bend loss coefficient in the third stage  $l_d/d = 19.5$

PIEZOMETER NO. ORIENTATION OF BEND DISCHARGE	B <sub>1</sub>		B <sub>2</sub>		B <sub>3</sub>		B <sub>4</sub>	
	H	V	H	V	H	V	H	V
0.04 cfs	1.95	54.25	-0.10	52.30	1.70	54.05	2.45	55.90
	1.90	54.20	-0.10	52.30	1.70	54.05	2.40	55.90
	2.00	54.25	-0.15	52.25	1.75	54.10	2.40	55.95
	1.90	54.25	-0.10	52.30	1.70	54.10	2.40	56.00
	2.20	54.45	-0.25	52.15	2.10	54.45	3.35	56.65
0.06 cfs	2.20	54.40	-0.25	52.10	2.10	54.40	3.40	56.60
	2.15	54.40	-0.20	52.15	2.15	54.45	3.40	56.60
	2.20	54.45	-0.20	52.10	2.10	54.40	3.30	56.65
	3.70	57.05	-1.35	51.80	3.70	57.15	5.75	59.10
	3.70	57.10	-1.40	51.85	3.75	57.20	5.75	59.10
0.10 cfs	3.75	57.05	-1.35	51.80	3.75	57.25	5.70	59.15
	3.65	57.15	-1.30	51.80	3.80	57.20	5.70	59.10
	5.90	60.80	-3.35	51.60	6.00	60.80	10.50	65.25
	5.95	60.70	-3.40	51.55	6.05	60.75	10.45	65.30
	5.90	60.65	-3.30	51.60	5.95	60.70	10.45	65.35
0.14 cfs	5.70	60.75	-3.35	51.50	5.90	60.90	10.60	65.40
	8.95	65.75	-5.55	51.00	9.25	66.25	18.25	72.85
	8.90	65.80	-5.50	51.00	9.30	66.20	18.20	72.90
	8.80	65.70	-5.60	51.05	9.35	66.10	18.20	72.90
	8.90	65.65	-5.50	50.90	9.10	66.15	18.15	72.70
0.18 cfs	12.45	72.00	-7.75	52.15	13.05	72.50	29.80	81.35
	12.50	72.05	-7.70	52.20	13.15	72.45	29.80	81.50
	12.50	72.20	-7.70	52.30	13.20	72.40	29.80	81.45
	16.05		-9.25		16.90		43.05	
	16.10		-9.20		16.75		43.00	
0.22 cfs	16.15		-9.20		16.80		43.15	
	16.20		-9.15		16.70		43.00	
0.26 cfs								

\* The data are the pressure heads, measured in inches, at the stations designated.



Test data\* for determining the bend loss coefficient in the fourth stage  $1_d/d = 13.5$

PIEZOMETER NO. ORIENTATION OF BEND DISCHARGE	B <sub>1</sub>		B <sub>2</sub>		B <sub>3</sub>		B <sub>4</sub>	
	H	V	H	V	H	V	H	V
0.04 cfs	1.80	41.05	0.10	38.20	1.70	41.10	2.40	41.65
	1.80	41.05	0.15	38.20	1.70	41.10	2.45	41.60
	1.75	41.10	0.15	38.25	1.75	41.10	2.45	41.60
	1.75	41.05	0.10	38.20	1.75	41.05	2.40	41.65
	1.90	41.35	-0.30	38.05	1.80	41.30	3.10	42.25
0.06 cfs	1.85	41.30	-0.25	38.10	1.85	41.30	3.15	42.20
	1.80	41.30	-0.25	38.10	1.85	41.35	3.20	42.20
	1.85	41.35	-0.30	38.10	1.80	41.30	3.15	42.25
	3.35	43.50	-0.95	37.40	3.40	43.55	6.55	45.65
	3.40	43.45	-0.90	37.45	3.45	43.55	6.40	45.60
0.10 cfs	3.45	43.45	-0.90	37.50	3.40	43.60	6.45	45.70
	3.40	43.40	-0.95	37.50	3.40	43.65	6.50	45.75
	4.90	46.90	-1.40	37.75	5.05	47.25	9.85	51.20
	4.95	46.90	-1.45	37.70	5.05	47.30	9.90	51.15
	4.95	46.95	-1.40	37.70	5.10	47.35	9.80	51.10
0.14 cfs	4.80	46.80	-1.35	37.65	5.15	47.20	9.75	51.10
	7.25	51.55	-2.00	36.30	7.55	52.15	14.70	58.75
	7.30	51.60	-2.00	36.20	7.70	52.20	14.65	58.75
	7.20	51.60	-2.20	36.30	7.65	52.10	14.65	58.60
	7.15	51.70	-2.10	36.15	7.65	52.10	14.50	58.65
0.18 cfs	10.20	56.55	-2.45	35.50	10.70	57.65	20.65	67.05
	10.15	56.60	-2.35	35.50	10.60	57.60	20.60	67.05
	10.20	56.50	-2.40	35.45	10.65	57.60	20.60	67.10
	10.05	56.50	-2.45	35.40	10.60	57.70	20.65	67.10
	12.40		-2.95		13.20		25.75	
0.22 cfs	12.45		-2.80		13.10		25.80	
	12.40		-2.75		13.15		25.85	
	12.20		-2.85		13.10		25.60	
0.26 cfs								

\* The data are the pressure heads, measured in inches, at the stations designated.

Test data\* for determining the bend loss coefficient in the fifth stage  $l_d/d = 1.5$

PIEZOMETER NO. ORIENTATION OF BEND DISCHARGE	$D_1$		$B_1$		$B_2$		$B_3$		$B_4$	
	H	V	H	V	H	V	H	V	H	V
0.04 cfs	0.80	16.20	1.05	16.35	-1.15	14.70	0.85	16.20	1.80	16.95
	0.85	16.20	1.00	16.35	-1.10	14.70	0.85	16.20	1.80	17.00
	0.80	16.15	1.00	16.40	-1.10	14.75	0.85	16.15	1.85	16.90
	0.80	16.20	1.00	16.40	-1.15	14.70	0.85	16.15	1.85	16.90
0.06 cfs	0.75	16.30	1.05	16.45	-1.30	14.30	0.95	16.35	1.95	17.35
	0.75	16.35	1.10	16.50	-1.25	14.25	0.90	16.40	1.90	17.30
	0.80	16.35	1.10	16.40	-1.30	14.20	0.95	16.40	1.95	17.30
	0.80	16.30	1.15	16.45	-1.35	14.20	0.95	16.30	1.90	17.35
0.10 cfs	1.45	18.05	1.75	18.25	-2.00	12.85	1.65	18.30	3.90	20.50
	1.50	18.10	1.85	18.30	-2.05	12.85	1.75	18.45	3.95	20.40
	1.45	18.10	1.70	18.35	-2.00	12.80	1.75	18.40	4.00	20.45
	1.50	18.20	1.75	18.25	-2.00	12.95	1.80	18.35	3.80	20.40
0.14 cfs	1.25	19.25	2.45	20.35	-2.85	11.45	2.40	20.55	6.40	24.55
	1.30	19.35	2.50	20.40	-2.70	11.40	2.40	20.60	6.55	24.40
	1.35	19.40	2.60	20.40	-2.75	11.45	2.45	20.65	6.45	24.45
	1.30	19.40	2.55	20.55	-2.70	11.40	2.40	20.75	6.40	24.50
0.18 cfs	3.05	23.35	3.60	23.80	-3.55	9.65	3.80	24.25	10.15	30.85
	3.05	23.45	3.65	23.80	-3.40	9.60	3.85	24.30	10.20	30.90
	3.20	23.40	3.80	23.85	-3.60	9.50	3.80	24.35	10.25	30.70
	3.10	23.40	3.70	23.90	-3.65	9.60	3.90	24.30	10.35	30.75
0.22 cfs	3.50	27.15	4.95	28.15	-4.15	6.60	5.45	28.60	14.80	38.55
	3.45	27.20	4.80	28.20	-4.20	6.65	5.50	28.75	14.85	38.50
	3.40	27.20	4.75	28.10	-4.10	6.60	5.50	28.75	14.80	38.50
	3.40	27.25	4.70	28.10	-4.20	6.70	5.30	28.80	14.90	38.75
0.26 cfs	4.10		6.50		-4.85		7.35		20.05	
	4.20		6.45		-4.70		7.40		20.05	
	4.20		6.40		-4.75		7.40		20.10	
	4.25		6.50		-4.70		7.35		20.20	

\* The data are the pressure heads, measured in inches, at the stations designated.

Appendix III Test data\* for determining the pressure distribution at a cross section

DISCHARGE	PIEZOMETER ANGLE OF TAP $\theta$	NO.	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$
0.06 cfs	0°		4.05	3.95	3.90	3.80	3.70
	90°		4.30	4.05	3.90	3.80	3.70
	180°		4.55	4.00	3.90	3.80	3.70
	270°		4.30	4.00	3.90	3.80	3.70
0.10 cfs	0°		4.85	4.25	4.00	3.75	3.40
	90°		5.00	4.35	4.00	3.75	3.40
	180°		5.50	4.50	4.00	3.70	3.40
	270°		5.00	4.30	4.05	3.70	3.40
0.14 cfs	0°		5.45	4.75	4.30	3.85	3.40
	90°		5.70	4.80	4.30	3.85	3.40
	180°		7.35	5.10	4.30	3.85	3.40
	270°		5.75	4.80	4.30	3.85	3.40
0.18 cfs	0°		5.45	5.50	4.75	4.05	3.30
	90°		6.90	5.65	4.75	4.05	3.30
	180°		9.35	6.00	4.75	4.05	3.30
	270°		7.00	5.65	4.75	4.05	3.30
0.22 cfs	0°		6.05	6.15	5.30	4.30	3.25
	90°		8.05	6.35	5.35	4.30	3.25
	180°		12.00	6.95	5.30	4.30	3.25
	270°		7.85	6.35	5.30	4.30	3.25
0.26 cfs	0°		6.25	7.80	5.75	4.50	3.20
	90°		9.25	7.00	5.75	4.50	3.20
	180°		14.20	6.55	5.75	4.50	3.20
	270°		8.90	7.00	5.75	4.50	3.20

\* The data are the pressure heads, measured in inches, at the stations designated.

LOSS OF PRESSURE IN WATER FLOWING THROUGH 90° PIPE BEND

by

CHEN-TZU YU

B. S., National Taiwan University, 1960

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AN ABSTRACT OF A MASTER'S THESIS

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For years, the study of the pressure loss produced by a bend has attracted the interest of many investigators. The work discussed in this thesis was undertaken to study the effects of downstream tangent, Reynolds number, orientation of bend (horizontal and vertical) and downstream transition length on bend loss coefficient. Pressure losses were determined for 2 in. PVC, 90 degree pipe bend having a 3.28 in. radius of curvature. The experimental results obtained show that the bend loss coefficient,  $K$ , depends on the position of the pressure measurements for  $x_d/d$  less than 36 and is independent of the position of the pressure measurements for  $x_d/d$  greater than 36. For short tangent having certain value of  $l_d/d$  (less than 12 diameters),  $K$  depends on both  $x_d/d$  and  $l_d/d$ .

This investigation also indicates that a reduction of  $K_G$  occurred for increasing Reynolds number  $N_R$ .